13-1 Right Triangle Trigonometry

Main Ideas

- Find values of trigonometric functions for acute angles.
- Solve problems involving right triangles.

New Vocabulary

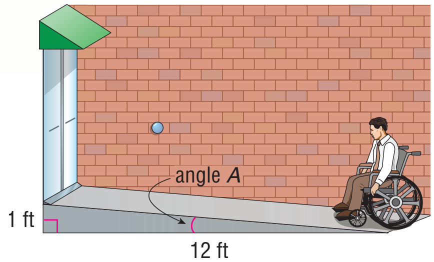
trigonometry
trigonometric functions
sine
cosine
tangent
cosecant
secant
cotangent
solve a right triangle
angle of elevation
angle of depression

Reading Math

Trigonometry
The word *trigonometry* is derived from two Greek words–*trigon* meaning triangle and *metra* meaning measurement.

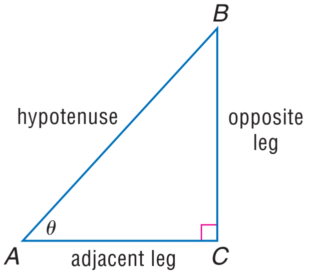
GET READY for the Lesson

The Americans with Disabilities Act (ADA) provides regulations designed to make public buildings accessible to all. Under this act, the slope of an entrance ramp designed for those with mobility disabilities must not exceed a ratio of 1 to 12. This means that for every 12 units of horizontal run, the ramp can rise or fall no more than 1 unit.



When viewed from the side, a ramp forms a right triangle. The slope of the ramp can be described by the *tangent* of the angle the ramp makes with the ground. In this example, the tangent of angle A is $\frac{1}{12}$.

Trigonometric Values The tangent of an angle is one of the ratios used in trigonometry. **Trigonometry** is the study of the relationships among the angles and sides of a right triangle.



Consider right triangle ABC in which the measure of acute angle A is identified by the Greek letter *theta*, θ. The sides of the triangle are the *hypotenuse*, the *leg opposite* θ, and the *leg adjacent to* θ.

Using these sides, you can define six **trigonometric functions: sine, cosine, tangent, cosecant, secant,** and **cotangent**. These functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.

KEY CONCEPT — *Trigonometric Functions*

If θ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite θ, *adj* is the measure of the leg adjacent to θ, and *hyp* is the measure of the hypotenuse, then the following are true.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Notice that the sine, cosine, and tangent functions are reciprocals of the cosecant, secant, and cotangent functions, respectively. Thus, the following are also true.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Study Tip

Memorize Trigonometric Ratios

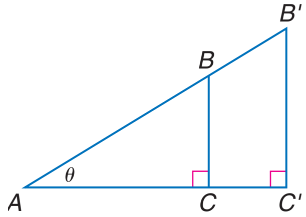
SOH-CAH-TOA is a mnemonic device for remembering the first letter of each word in the ratios for sine, cosine, and tangent.

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

The domain of each of these trigonometric functions is the set of all acute angles θ of a right triangle. The values of the functions depend only on the measure of θ and not on the size of the right triangle. For example, consider $\sin\theta$ in the figure at the right.



Using $\triangle ABC$:

$$\sin\theta = \frac{BC}{AB}$$

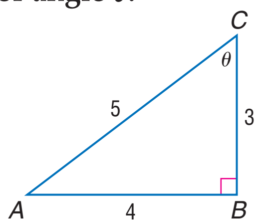
Using $\triangle AB'C'$:

$$\sin\theta = \frac{B'C'}{AB'}$$

The right triangles are similar because they share angle θ. Since they are similar, the ratios of corresponding sides are equal. That is, $\frac{BC}{AB} = \frac{B'C'}{AB'}$. Therefore, you will find the same value for $\sin\theta$ regardless of which triangle you use.

EXAMPLE — Find Trigonometric Values

1 **Find the values of the six trigonometric functions for angle θ.**



For this triangle, the leg opposite θ is $\overline{AB}$, and the leg adjacent to θ is $\overline{CB}$. Recall that the hypotenuse is always the longest side of a right triangle, in this case $\overline{AC}$.

Use opp = 4, adj = 3, and hyp = 5 to write each trigonometric ratio.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

CHECK Your Progress

1. Find the values of the six trigonometric functions for angle A in $\triangle ABC$ above.

Throughout Unit 5, a capital letter will be used to denote both a vertex of a triangle and the measure of the angle at that vertex. The same letter in lowercase will be used to denote the side opposite that angle and its measure.

STANDARDIZED TEST EXAMPLE — Use One Trigonometric Ratio to Find Another

2 **If $\cos A = \frac{2}{5}$, find the value of $\tan A$.**

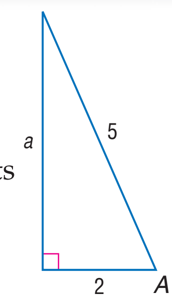
A $\frac{5}{2}$ **B** $\frac{2\sqrt{21}}{21}$ **C** $\frac{\sqrt{21}}{2}$ **D** $\sqrt{21}$

Test-Taking Tip

Whenever necessary or helpful, draw a diagram of the situation.

Read the Test Item

Begin by drawing a right triangle and labeling one acute angle A. Since $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ and $\cos A = \frac{2}{5}$ in this case, label the adjacent leg 2 and the hypotenuse 5. This represents the simplest triangle for which $\cos A = \frac{2}{5}$.



Solve the Test Item

Use the Pythagorean Theorem to find a.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$a^2 + 2^2 = 5^2$ Replace b with 2 and c with 5.

$a^2 + 4 = 25$ Simplify.

$a^2 = 21$ Subtract 4 from each side.

$a = \sqrt{21}$ Take the square root of each side.

Now find tan A.

$\tan A = \dfrac{\text{opp}}{\text{adj}}$ Tangent ratio

$= \dfrac{\sqrt{21}}{2}$ Replace *opp* with $\sqrt{21}$ and *adj* with 2.

The answer is C.

CHECK Your Progress

2. If $\tan B = \frac{3}{7}$, find the value of sin B.

F $\frac{7}{3}$ **G** $\frac{\sqrt{58}}{3}$ **H** $\frac{3\sqrt{58}}{58}$ **J** $\frac{\sqrt{58}}{7}$

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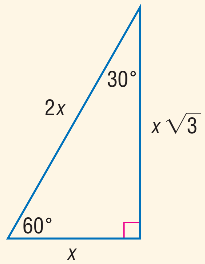
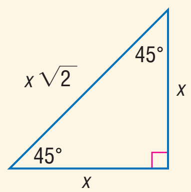
Angles that measure 30°, 45°, and 60° occur frequently in trigonometry. The table below gives the values of the six trigonometric functions for these angles. To remember these values, use the properties of 30°-60°-90° and 45°-45°-90° triangles.

KEY CONCEPT *Trigonometric Values for Special Angles*

30°-60°-90° Triangle



45°-45°-90° Triangle



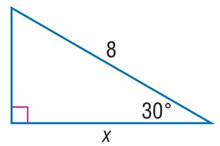
θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

You will verify some of these values in Exercises 39 and 40.

Right Triangle Problems You can use trigonometric functions to solve problems involving right triangles.

EXAMPLE 3 Find a Missing Side Length of a Right Triangle

Write an equation involving sin, cos, or tan that can be used to find the value of x. Then solve the equation. Round to the nearest tenth.



The measure of the hypotenuse is 8. The side with the missing length is *adjacent* to the angle measuring 30°. The trigonometric function relating the adjacent side of a right triangle and the hypotenuse is the cosine function.

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$ cosine ratio

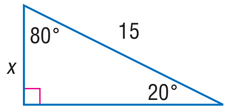
$$\cos 30° = \frac{x}{8}$$ Replace θ with 30°, *adj* with x, and *hyp* with 8.

$$\frac{\sqrt{3}}{2} = \frac{x}{8}$$ $\cos 30° = \frac{\sqrt{3}}{2}$

$$4\sqrt{3} = x$$ Multiply each side by 8. The value of x is $4\sqrt{3}$ or about 6.9.

CHECK Your Progress

3. Write an equation involving sin, cos, or tan that can be used to find the value of x. Then solve the equation. Round to the nearest tenth.



> **Study Tip**
>
> **Common Misconception**
>
> The $\cos^{-1} x$ on a graphing calculator does *not* find $\frac{1}{\cos x}$. To find sec x or $\frac{1}{\cos x}$, find cos x and then use the $\boxed{x^{-1}}$ key.

A calculator can be used to find the value of trigonometric functions for *any* angle, not just the special angles mentioned. Use $\boxed{\text{SIN}}$, $\boxed{\text{COS}}$, and $\boxed{\text{TAN}}$ for sine, cosine, and tangent. Use these keys and the reciprocal key, $\boxed{x^{-1}}$, for cosecant, secant, and cotangent. Be sure your calculator is in degree mode.

Here are some calculator examples.

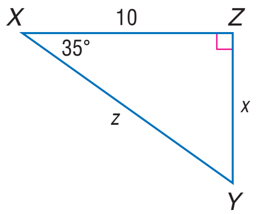
cos 46° **KEYSTROKES:** $\boxed{\text{COS}}$ 46 $\boxed{\text{ENTER}}$ 0.6946583705

cot 20° **KEYSTROKES:** $\boxed{\text{TAN}}$ 20 $\boxed{\text{ENTER}}$ $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$ 2.747477419

If you know the measures of any two sides of a right triangle or the measures of one side and one acute angle, you can determine the measures of all the sides and angles of the triangle. This process of finding the missing measures is known as **solving a right triangle**.

EXAMPLE 4 Solve a Right Triangle

Solve $\triangle XYZ$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



Find x and z.

$$\tan 35° = \frac{x}{10} \qquad \sec 35° = \frac{z}{10}$$

$$10 \tan 35° = x \qquad \frac{1}{\cos 35°} = \frac{z}{10}$$

$$7.0 \approx x \qquad \frac{1}{\cos 35°} = z$$

$$12.2 \approx z$$

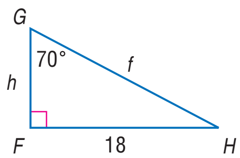
Find Y. $35° + Y = 90°$ Angles X and Y are complementary.

$Y = 55°$ Therefore, $Y = 55°$, $x \approx 7.0$, and $z \approx 12.2$.

> **Study Tip**
>
> **Error in Measurement**
>
> The value of z in Example 4 is found using the secant instead of using the Pythagorean Theorem. This is because the secant uses values given in the problem rather than calculated values.

CHECK Your Progress

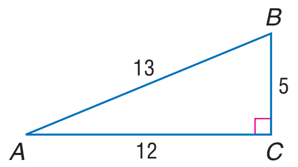
4. Solve $\triangle FGH$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



Use the inverse capabilities of your calculator to find the measure of an angle when one of its trigonometric ratios is known. For example, use the $\sin^{-1}$ function to find the measure of an angle when the sine of the angle is known. *You will learn more about inverses of trigonometric functions in Lesson 13-7.*

EXAMPLE Find Missing Angle Measures of Right Triangles

5 **Solve $\triangle ABC$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**



You know the measures of the sides. You need to find A and B.

Find A. $\quad \sin A = \frac{5}{13}$ $\quad$ $\sin A = \frac{\text{opp}}{\text{hyp}}$

Use a calculator and the [SIN^{-1}] function to find the angle whose sine is $\frac{5}{13}$.

KEYSTROKES: 2nd [SIN^{-1}] 5 ÷ 13) ENTER 22.61986495

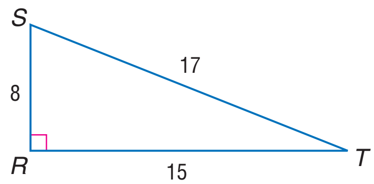
To the nearest degree, $A \approx 23°$.

Find B. $\quad 23° + B \approx 90°$ $\quad$ Angles A and B are complementary.

$B \approx 67°$ $\quad$ Solve for B.

Therefore, $A \approx 23°$ and $B \approx 67°$.

CHECK Your Progress



5. Solve $\triangle RST$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

Trigonometry has many practical applications. Among the most important is the ability to find distances that either cannot or are not easily measured directly.

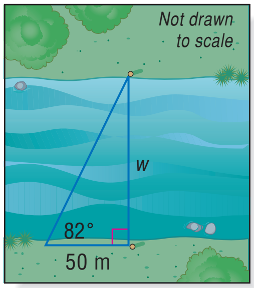
Real-World EXAMPLE Indirect Measurement

Real-World Link

There are an estimated 595,625 bridges in use in the United States.

Source: betterroads.com

6 **BRIDGE CONSTRUCTION** **In order to construct a bridge, the width of the river must be determined. Suppose a stake is planted on one side of the river directly across from a second stake on the opposite side. At a distance 50 meters to the left of the stake, an angle of 82° is measured between the two stakes. Find the width of the river.**



Let w represent the width of the river at that location. Write an equation using a trigonometric function that involves the ratio of the distance w and 50.

$\tan 82° = \frac{w}{50}$ $\quad$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$50 \tan 82° = w$ $\quad$ Multiply each side by 50.

$355.8 \approx w$ $\quad$ The width of the river is about 355.8 meters.

CHECK Your Progress

6. John found two trees directly across from each other in a canyon. When he moved 100 feet from the tree on his side (parallel to the edge of the canyon), the angle formed by the tree on his side, John, and the tree on the other side was 70°. Find the distance across the canyon.

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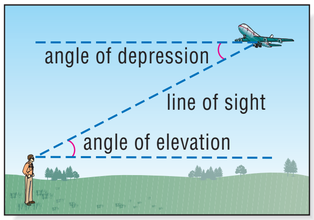
Study Tip

Angle of Elevation and Depression

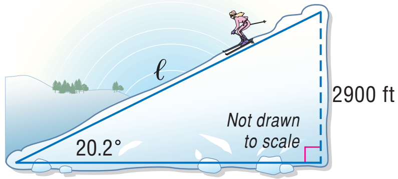
The angle of elevation and the angle of depression are congruent since they are alternate interior angles of parallel lines.

Some applications of trigonometry use an angle of elevation or depression. In the figure at the right, the angle formed by the line of sight from the observer and a line parallel to the ground is called the **angle of elevation.** The angle formed by the line of sight from the plane and a line parallel to the ground is called the **angle of depression.**



EXAMPLE Use an Angle of Elevation

7 **SKIING** **The Aerial run in Snowbird, Utah, has an angle of elevation of 20.2°. Its vertical drop is 2900 feet. Estimate the length of this run.**



Let ℓ represent the length of the run. Write an equation using a trigonometric function that involves the ratio of ℓ and 2900.

$\sin 20.2° = \dfrac{2900}{\ell}$ $\sin \theta = \dfrac{\text{opp}}{\text{hyp}}$

$\ell = \dfrac{2900}{\sin 20.2°}$ Solve for ℓ.

$\ell \approx 8398.5$ Use a calculator.

The length of the run is about 8399 feet.

Real-World Link

The average annual snowfall in Alpine Meadows, California, is 495 inches. The longest designated run there is 2.5 miles.

Source: www.onthesnow.com

CHECK Your Progress

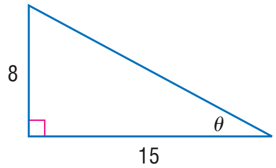
7. A ramp for unloading a moving truck has an angle of elevation of 32°. If the top of the ramp is 4 feet above the ground, estimate the length of the ramp.

CHECK Your Understanding

Example 1 (p. 760)

Find the values of the six trigonometric functions for angle θ.

1.



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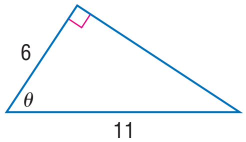

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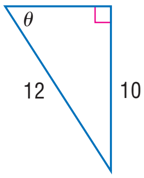


Example 2 (pp. 760–761)

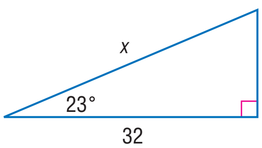
4. **STANDARDIZED TEST PRACTICE** If $\tan \theta = 3$, find the value of $\sin \theta$. **B**

A $\dfrac{3}{10}$ B $\dfrac{3\sqrt{10}}{10}$ C $\dfrac{10}{3}$ D $\dfrac{1}{3}$

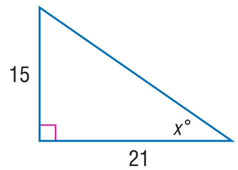
Examples 3, 5 (pp. 761–763)

Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and angles to the nearest degree.

5.



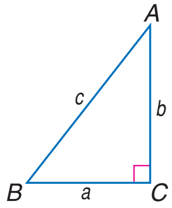
6.



John P. Kelly/Getty Images

Examples 4, 5 (pp. 762–763)

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



7. $A = 45°, b = 6$

8. $B = 56°, c = 6$

9. $b = 7, c = 18$

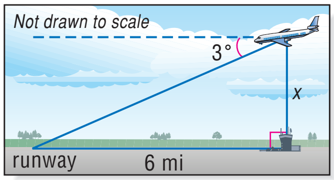
10. $a = 14, b = 13$

Example 6 (p. 763)

11. BRIDGES Tom wants to build a rope bridge between his tree house and Roy's tree house. Suppose Tom's tree house is directly behind Roy's tree house. At a distance of 20 meters to the left of Tom's tree house, an angle of 52° is measured between the two tree houses. Find the length of the rope bridge.

Example 7 (p. 764)

12. AVIATION When landing, a jet will average a 3° angle of descent. What is the altitude x, to the nearest foot, of a jet on final descent as it passes over an airport beacon 6 miles from the start of the runway?

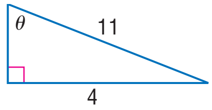


Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–14	1, 2
15–18	3
21–26	4
19, 20	5
27, 28	6, 7

Find the values of the six trigonometric functions for angle θ.

13.



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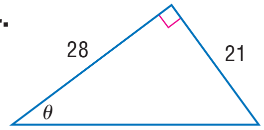

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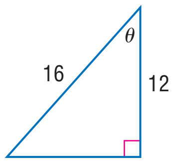
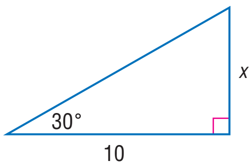


Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and angles to the nearest degree.

16.



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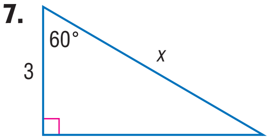

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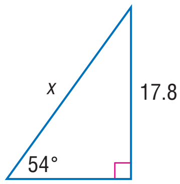

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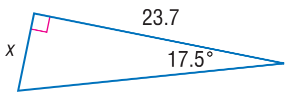

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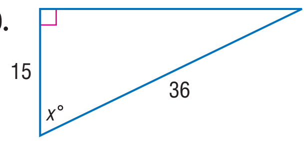

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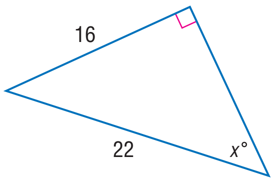

Real-World Career

Surveyor

Land surveyors manage survey parties that measure distances, directions, and angles between points, lines, and contours on Earth's surface.

Math Online

For more information, go to algebra2.com.

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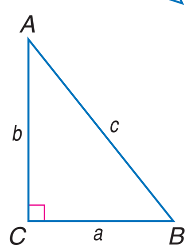
Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



22. $A = 16°, c = 14$

23. $B = 27°, b = 7$

24. $A = 34°, a = 10$

25. $B = 15°, c = 25$

26. $B = 30°, b = 11$

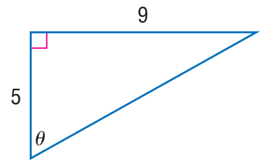
27. $A = 45°, c = 7\sqrt{2}$

28. SURVEYING A surveyor stands 100 feet from a building and sights the top of the building at a 55° angle of elevation. Find the height of the building.

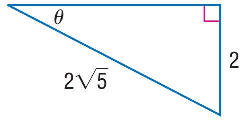
29. **TRAVEL** In a sightseeing boat near the base of the Horseshoe Falls at Niagara Falls, a passenger estimates the angle of elevation to the top of the falls to be 30°. If the Horseshoe Falls are 173 feet high, what is the distance from the boat to the base of the falls?

Find the values of the six trigonometric functions for angle θ.

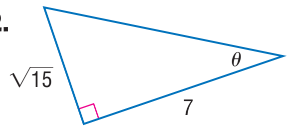
30.



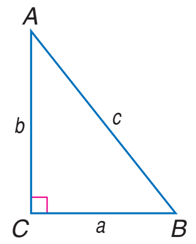
31.



32.



Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



33. $B = 18°, a = \sqrt{15}$

34. $A = 10°, b = 15$

35. $b = 6, c = 13$

36. $a = 4, c = 9$

37. $\tan B = \frac{7}{8}, b = 7$

38. $\sin A = \frac{1}{3}, a = 5$

39. Using the 30°-60°-90° triangle shown in the lesson, verify each value.

a. $\sin 30° = \frac{1}{2}$ **b.** $\cos 30° = \frac{\sqrt{3}}{2}$ **c.** $\sin 60° = \frac{\sqrt{3}}{2}$

40. Using the 45°-45°-90° triangle shown in the lesson, verify each value.

a. $\sin 45° = \frac{\sqrt{2}}{2}$ **b.** $\cos 45° = \frac{\sqrt{2}}{2}$ **c.** $\tan 45° = 1$

Cross-Curricular Project

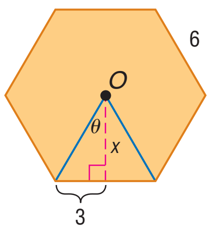
Math Online You can use the tangent ratio to determine the maximum height of a rocket. Visit algebra2.com to continue work on your project.

EXERCISE For Exercises 41 and 42, use the following information.
A preprogrammed workout on a treadmill consists of intervals walking at various rates and angles of incline. A 1% incline means 1 unit of vertical rise for every 100 units of horizontal run.

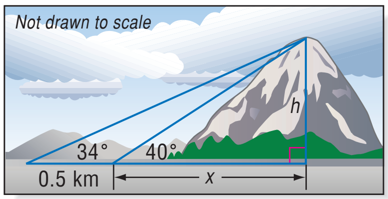
41. At what angle, with respect to the horizontal, is the treadmill bed when set at a 10% incline? Round to the nearest degree.

42. If the treadmill bed is 40 inches long, what is the vertical rise when set at an 8% incline?

43. **GEOMETRY** Find the area of the regular hexagon with point O as its center. (*Hint*: First find the value of x.)



44. **GEOLOGY** A geologist measured a 40° of elevation to the top of a mountain. After moving 0.5 kilometer farther away, the angle of elevation was 34°. How high is the top of the mountain? (*Hint*: Write a system of equations in two variables.)



EXTRA PRACTICE
See pages 920, 938.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

45. OPEN ENDED Draw two right triangles $\triangle ABC$ and $\triangle DEF$ for which $\sin A = \sin D$. What can you conclude about $\triangle ABC$ and $\triangle DEF$? Justify your reasoning.

46. REASONING Find a counterexample to the statement *It is always possible to solve a right triangle.*

47. CHALLENGE Explain why the sine and cosine of an acute angle are never greater that 1, but the tangent of an acute angle may be greater than 1.

48. *Writing in Math* Use the information on page 759 to explain how trigonometry is used in building construction. Include an explanation as to why the ratio of vertical rise to horizontal run on an entrance ramp is the tangent of the angle the ramp makes with the horizontal.

STANDARDIZED TEST PRACTICE

49. ACT/SAT If the secant of angle θ is $\frac{25}{7}$, what is the sine of angle θ?

A $\frac{5}{25}$

B $\frac{7}{25}$

C $\frac{24}{25}$

D $\frac{25}{7}$

50. REVIEW A person holds one end of a rope that runs through a pulley and has a weight attached to the other end. Assume the weight is directly beneath the pulley. The section of rope between the pulley and the weight is 12 feet long. The rope bends through an angle of 33 degrees in the pulley. How far is the person from the weight?

F 7.8 ft

G 10.5 ft

H 12.9 ft

J 14.3 ft

Spiral Review

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer (Lesson 12-9)

51. surveying band members to find the most popular type of music at your school

52. surveying people coming into a post office to find out what color cars are most popular

Find each probability if a coin is tossed 4 times (Lesson 12-8)

53. P(exactly 2 heads)

54. P(4 heads)

55. P(at least 1 head)

Solve each equation (Lesson 6-6)

56. $y^4 - 64 = 0$

57. $x^5 - 5x^3 + 4x = 0$

58. $d + \sqrt{d} - 132 = 0$

GET READY for the Next Lesson

PREREQUISITE SKILL **Find each product. Include the appropriate units with your answer.** (Lesson 6-1)

59. 5 gallons $\left(\frac{\text{4 quarts}}{\text{1 gallon}}\right)$

60. 6.8 miles $\left(\frac{\text{5280 feet}}{\text{1 mile}}\right)$

61. $\left(\frac{\text{2 square meters}}{\text{5 dollars}}\right)$30 dollars

62. $\left(\frac{\text{4 liters}}{\text{5 minutes}}\right)$60 minutes